Prediction of some random processes

Alexander V. Kramarenko

Many years ago, freedom and democracy together with civilization came to the city of Kharkov, which then was a large scientific and industrial center. Playing machines - primitive one-armed bandits - were placed at the railway station. Naturally, the people, still immaculate in their naivety, lost all their cash, including that which was necessary to pay the fare. The success of the owners of the machines was extraordinary: the money of passengers flowed by the river.

However, after a very short time, the machines began to lose, and their owners suffered losses. It was not right - by definition people should always lose, and the police got involved. Quite quickly it became clear that if you make a simple device with the same as the one-armed bandit chip random number generator, then after a few losses can synchronize the generators, and then the game will go with accurate prediction, and the bandit has no chance. “Attackers” with the generator, of course, were caught and put in jail, although they were only doing the same thing as millions of players on the stock exchange, who nevertheless do not go to jail.

Of course, this example has nothing to do with the actual analysis of a random process and is merely given to illustrate the benefits of effective prediction.

The world's first real working predictor was an instrument designed by the great William Thomson (Lord Kelvin). In this analyzer, the mechanical (!) calculation of Fourier harmonics provided accurate prediction of tide levels on the US coast for many years.

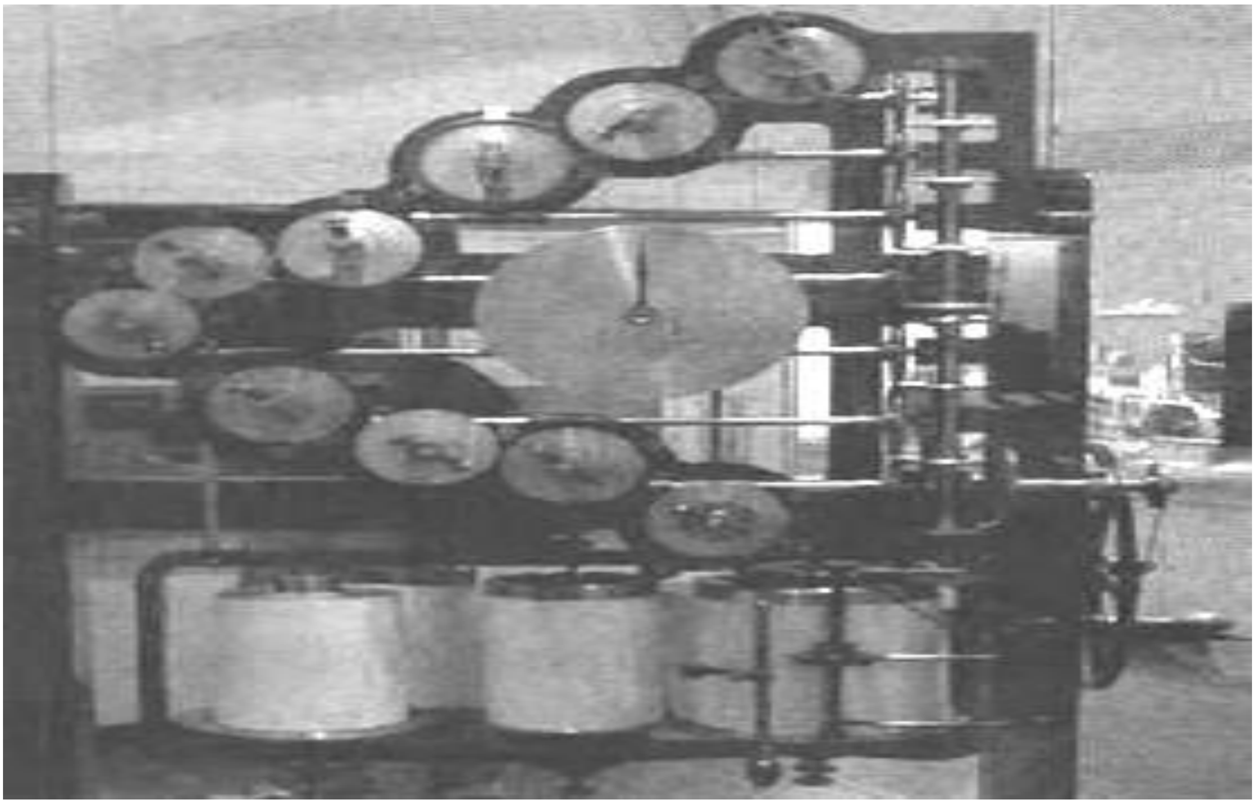

Fig.1. Harmonic tide analyzer on display at the Science Museum in London.

Tides, however, are not known to be a completely random process, and the author was clearly trying to develop the method. “In his lectures, Kelvin often showed his audience fanciful constructions of various elements (steel cables, pendulums, wooden grids with weights on the end, flywheels, bars, springs, and so on). The individual behavior of each element was well known, but all together they produced an infinite number of states of motion that were quite difficult to calculate” [1].

Note that this was exactly the generator of a random potentially predictable process. Kelvin was clearly trying to find a solution for which he did not have a powerful enough computational technique.

The modern situation with analysis and prediction of random and not quite random processes is very diverse, if I may put it this way: there are both very complicated works with serious and difficult to understand mathematics and attempts to reanimate Kondratiev's cycles, Slutsky's cycles and similar nonsense. The results of applying new and old methods are also different; predictions of such processes as, for example, economic activity, have not been very successful so far.

Many years ago, reanimatologists in CCB-5 in Kharkov wished to have a graph of some physiologic parameter for selection of drugs in the process of intensive therapy, which shows the process “forward”. The idea was clearly ahead of time and was right - it was a forecast “at the tip of the needle”, and it was planned that the results would be received in real time, and it would allow to correct therapy in time.

Such parameter turned out to be red shift in the energy spectrum of PQRST complexes of electrocardiogram. That is, as the generator (heart muscle) mismatches with the antenna-feeder device (vascular system), the VSWR (standing wave coefficient) increases, the load on the heart muscle increases, and the blood supply deteriorates: in this case, the energy spectrum of the ECG complex shifts to the red region (the effect is almost invisible visually). Of course, by administering medications and correction of electrolyte balance resuscitators seek to compensate for the developing insufficiency of blood supply. And if the redshift prognosis changes after drug administration, then corrections in treatment tactics can be made not only in time, but even preemptively, which is extremely important for resuscitation. But it is precisely the prediction of the behavior of the redshift curve that is necessary for preemptive work. A special difficulty lies in the fact that this curve changes both under the influence of physiologic processes and under the influence of pathologic changes and, of course, drug therapy.

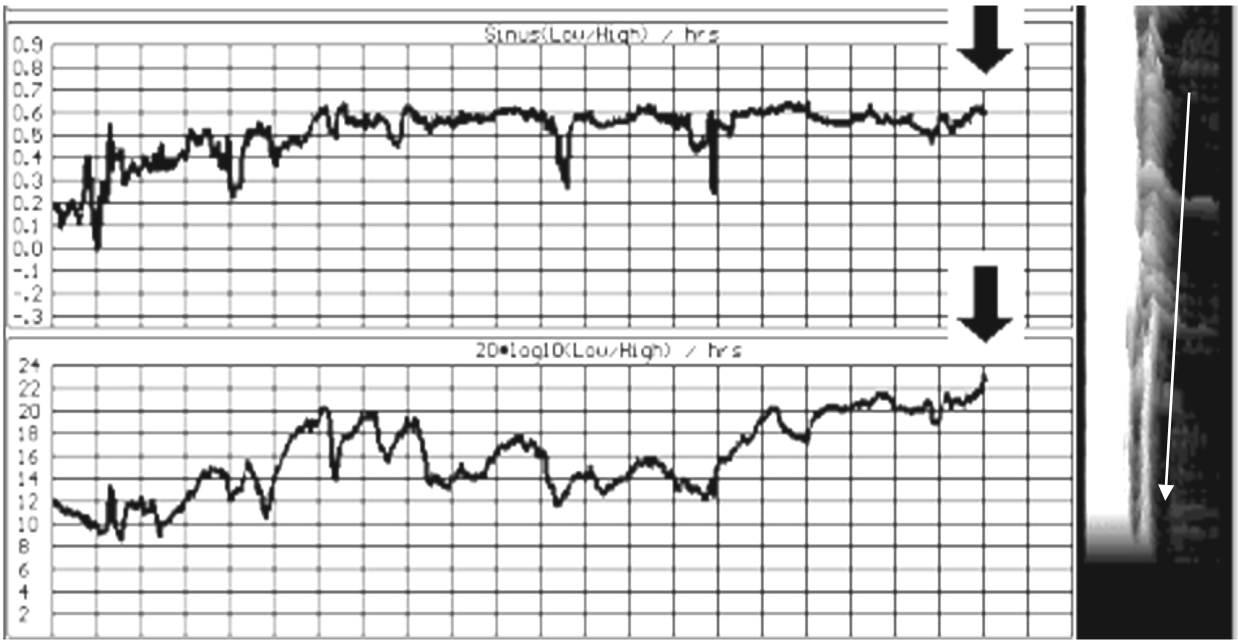

Fig.2. Red shift dynamics graph. Acute transmural anterior septal-apical myocardial infarction with formation of an acute aneurysm with wall thrombus. The cause of death (this moment is marked with an arrow) - linear rupture of the anterior wall of the left ventricle with hemotamponade of the heart. One can clearly see how the high-frequency components disappear on the spectrogram on the right, (early spectra are higher than late spectra).

This project after labor-intensive theoretical and experimental works took place, and the results were even better than expected. Most importantly, the method was approved by resuscitators.

However, the work was not developed: the enthusiasts left for more promising countries, and state structures were not interested in it.

In general - the usual story of intellectual involution. Traces remained only in the form of an article [2].

Still, I will allow myself to cite only one of many and many examples of predictions, which are almost impossible to believe, and natural skepticism appears even among the most benevolent colleagues.

The illustrations show the results of predicting the magnitude of the redshift. The system knows the behavior of the graph only up to (NB!) the gray vertical line in the figure.

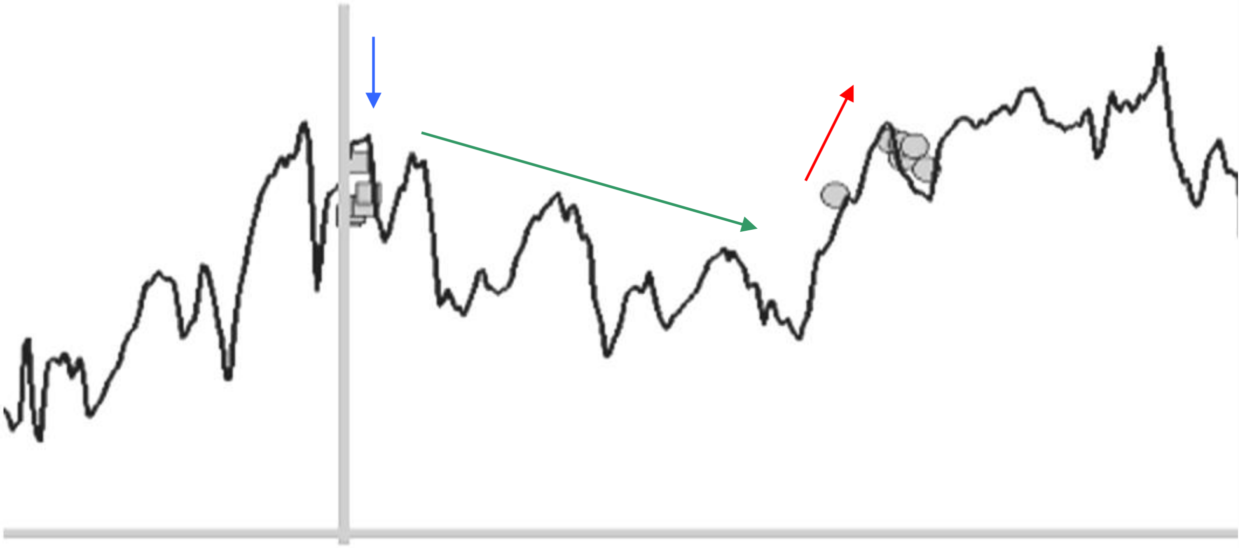

Fig.3. The same case of severe cardiac pathology, 22 hours of observation. Square marks indicate short-term (30 minutes) prognosis, long-term (12 hours) - round marks. At the moment indicated by the blue arrow, additional drug therapy was started: clinically, there was a significant improvement (green arrow), but the long-term prognosis does not confirm this.

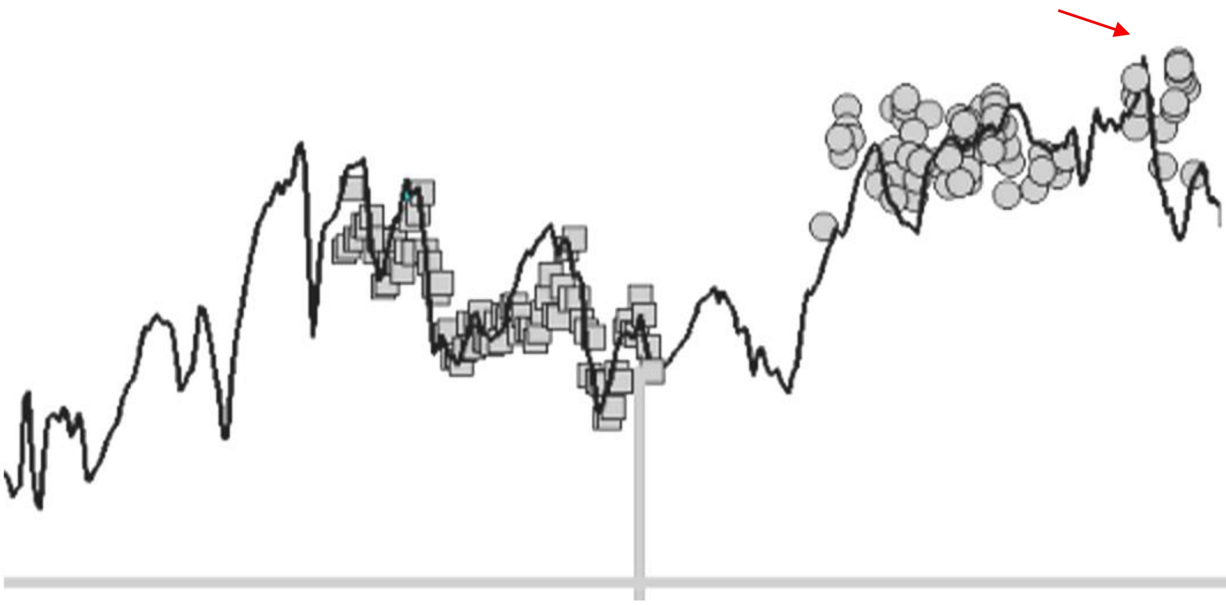

Fig.4. Situation after 7 hours. Still the system knows the curve only up to (NB!) the gray vertical line. The red arrow shows the predicted critical rise of the graph, while according to all clinical and hardware data the patient's condition was markedly improving under the influence of therapy. The catastrophe was predicted 12 hours before its manifestation [3].

One can, of course, assume that in the presented cases there is a deterministic process and the predictions do not present any difficulty. However, if there is a clear response of the curve to the introduction of various drugs, the invariance of the process is excluded (alas, it took a long time to realize this).

But let's leave covid and paracetamol mutilated medicine and return to “bizarre constructions of various elements (steel cables, pendulums...)”. Let us make an assumption that if the number of “cables and pendulums” rushes to a very large number, and in the limit - infinity, we can get a not quite polyharmonic process [4], which does not differ from the true noise. That is, this signal may even have a uniform spectrum and a normal Gaussian distribution, but will be predictable Kelvin noise (we think such a name would be appropriate out of respect for the great scientist).

In fact, the sum of a large number of Kelvin-Taylor instabilities would be, for example, such a noise [5]. Kelvin noises are also bound to exist within the framework of the hypothesis proposed by Gray-Walter: “stabilization through fluctuations”, which has been confidently confirmed in neurophysiology. (The alpha rhythm of the electroencephalogram, for example, is a nearly monochromatic signal with periodic beats, and its frequency coincides exactly with the limiting rate of recognition of changing images seen by humans - the frame sweep of the visual analyzer).

So, let there be a signal that is an “oscillatory mix”. Note that if we knew the oscillatory characteristics of oscillators, their number and power, i.e. the contribution of each to the resulting signal, this signal can be repeated (modeled) absolutely accurately. And if it is modeled with, a time advance relative to the main oscillator, then each count can be accurately predicted with any desired advance (repeating the history with one-armed bandits).

If the characteristics of the oscillators are unknown, it is possible to transform the signal into the frequency domain and then perform an inverse transformation with the synthesis of the signal forward, i.e. provide prediction in the time domain.

At the same time, a contradiction arises: “Non-singular processes include those for which it is impossible to trace the nature of cause-and-effect relations, since they are the result of superposition of a large number of elementary processes. It is fundamentally impossible to predict instantaneous values for them” [6]. I.e., in accordance with the basic provisions of modern theory it is impossible to collect predictable noise from a large number of elementary monochromatic signals.

It is possible, of course, that under elementary processes we mean some delta (?) pulses that have no representation in the frequency (NB!) domain. Then the question of the physical realizability of these components arises, and the specter of a bandwidth pointing to infinity alarms the imagination.

It is more probable, however, that physically realizable processes propagating along one communication channel, however close to white noise, cannot be such that cannot be represented in the frequency domain. For unpredictability of these processes it would be necessary to have infinite bandwidth and, accordingly, infinite power of the signal source. If, however, the bandwidth is limited and the Kotelnikov condition (Nyquist velocities) is satisfied when the signal is sampled, then prediction, at least with a large error and very small anticipation, must exist (an illustration of the fallacy of this hypothesis as well will be shown below).

It is useful to confirm theoretical considerations as well as one's own algorithms and programs by experiment. Therefore, we first synthesize known signals and check their spectrum and distribution function.

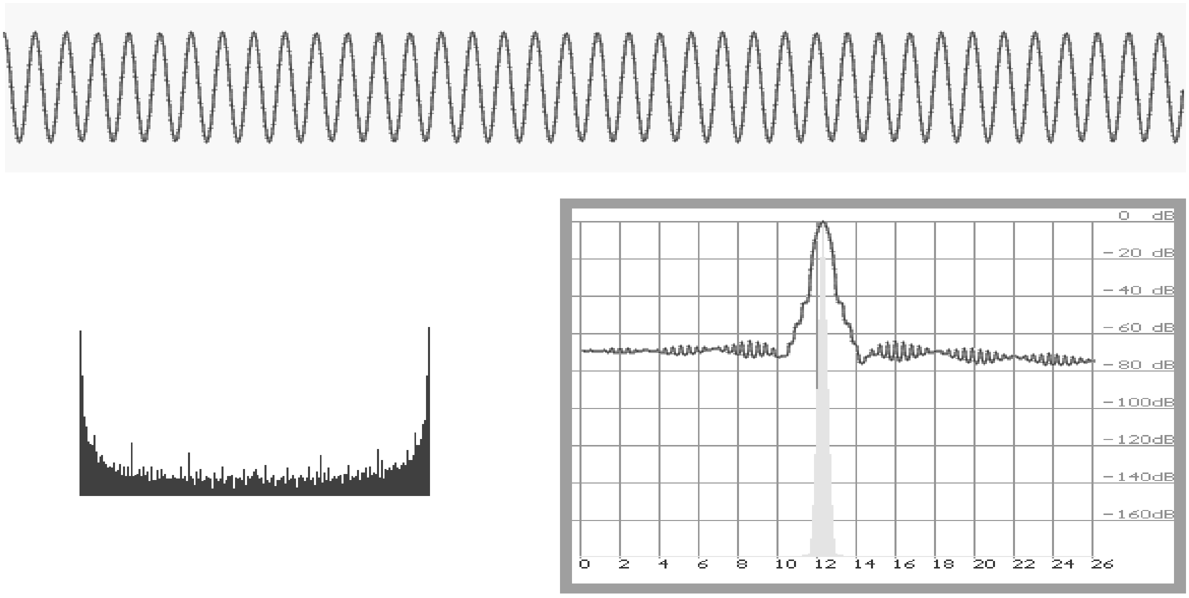

Fig.5. Monochromatic signal, its distribution function and spectrum (Hamming window is used).

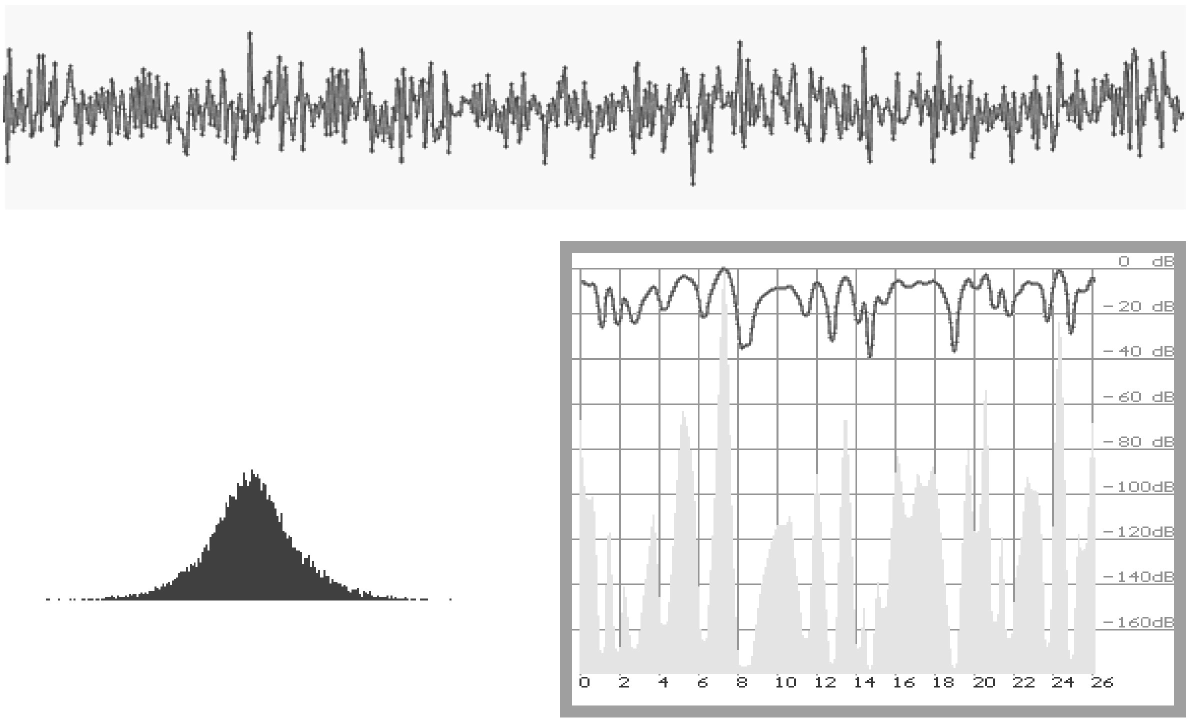

Fig.6. Normally distributed white Gaussian noise (the spectrum is calculated for a short realization and therefore the non-uniformity of the amplitude-frequency response of the selected fragment can be seen).

The presented simulation results are quite consistent with the basic theory, i.e. we can expect that the algorithm and the program are workable.

Now let us synthesize the signal that Kelvin wanted to obtain - an additive mixture of various generators of monochromatic signals with random initial phase. Note that the mutual influence of oscillators in the time domain will not matter for the resulting spectrum, since the total energy of the “system of cables and pendulums” will remain almost constant at a sufficiently high goodness of fit.

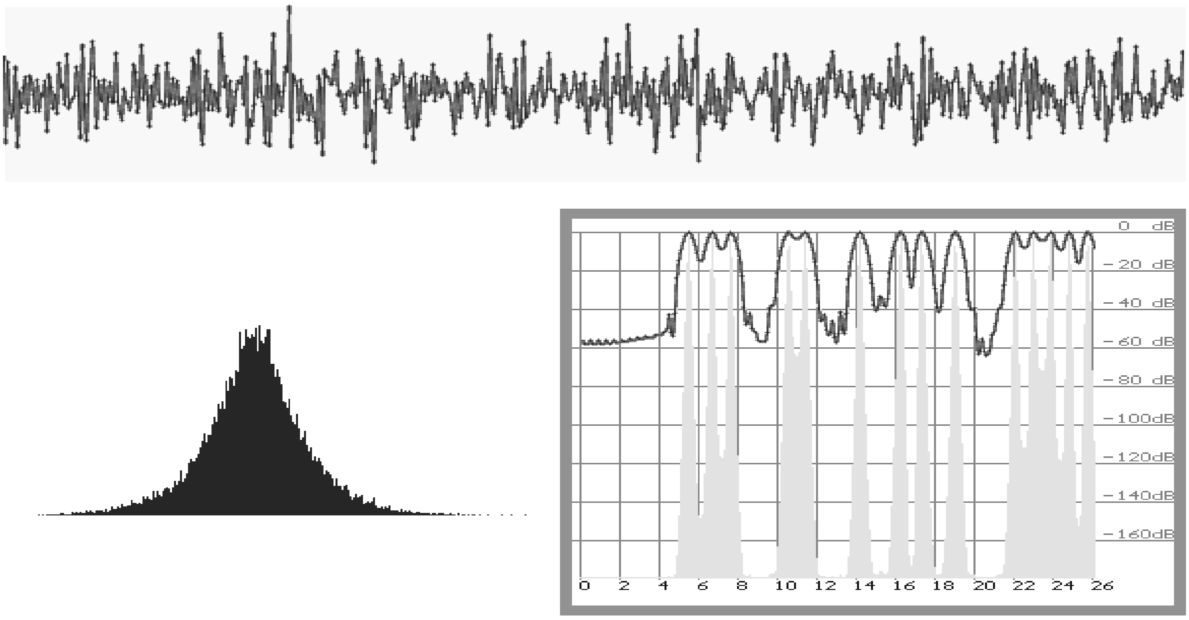

Fig.7. Noise signal resulting from the summation of monochromatic spectral components with arbitrary initial phase.

This signal can be expected to be completely predictable, but it should also be taken into account that beats of additive monochromatic processes will necessarily lead to periodicity of the resulting sequence. The periodicity may tend to very large values, but it will necessarily be there.

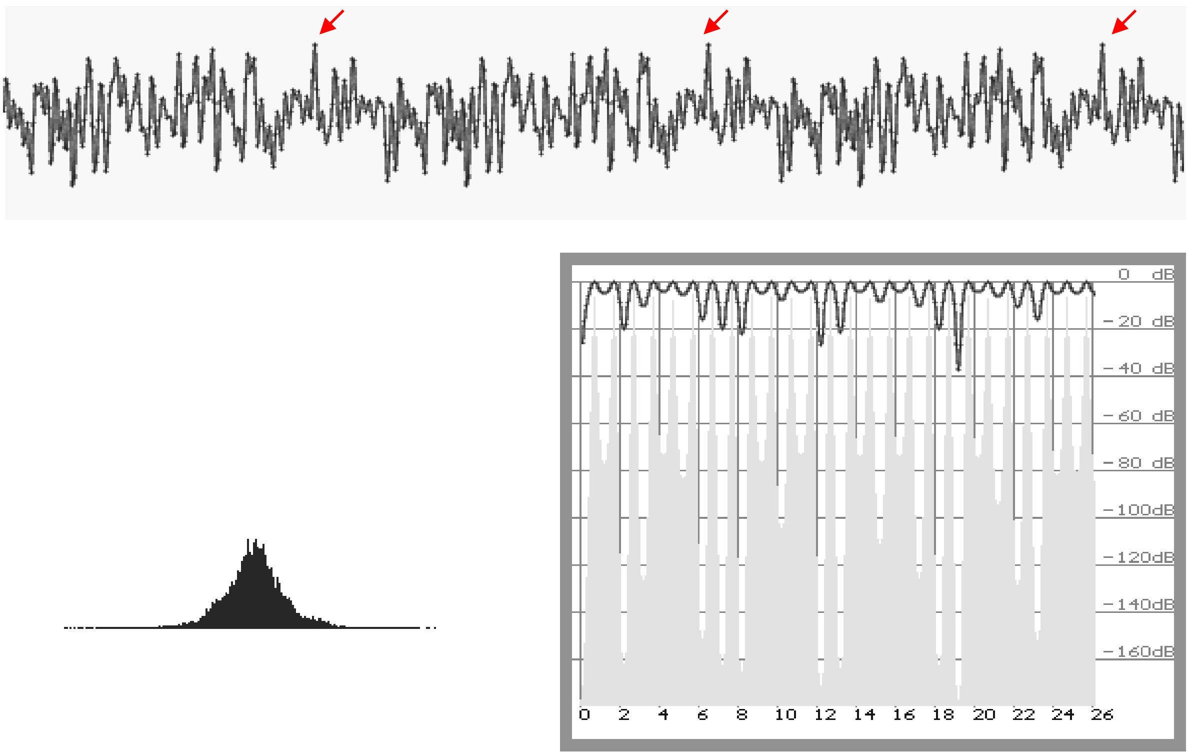

Fig.8. Illustration of a predictable noise signal with a uniform spectrum. The unfortunate frequencies of the resonators are chosen deliberately - in the time domain, an effect similar to the beautiful Kelvin-Helmholtz instability is obtained. The repetition period is indicated by arrows

Obviously, the algorithms for synthesizing predictable noise and the predictor itself are so simple that any reviewer of a scientific journal (not to mention an editor) can program them in half an hour. But for control purposes, let's check the possibility of efficient prediction of a monochromatic signal - there is no doubt about the theory here, but it is never superfluous to check the program module.

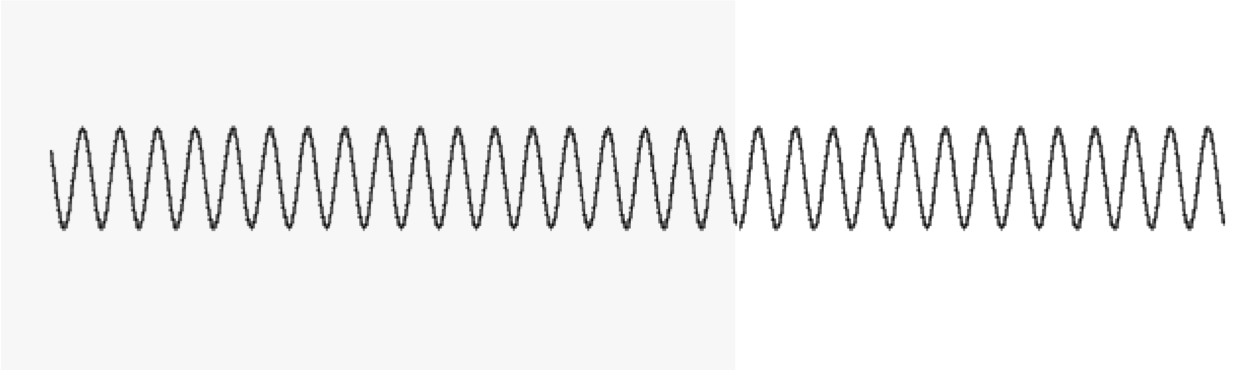

Fig.9. Let there be a harmonic signal known to the analyzer only within the left field highlighted in gray. We solve the problem of signal continuation while preserving its amplitude, frequency and phase. The moment of transition to prediction is shown by the arrow.

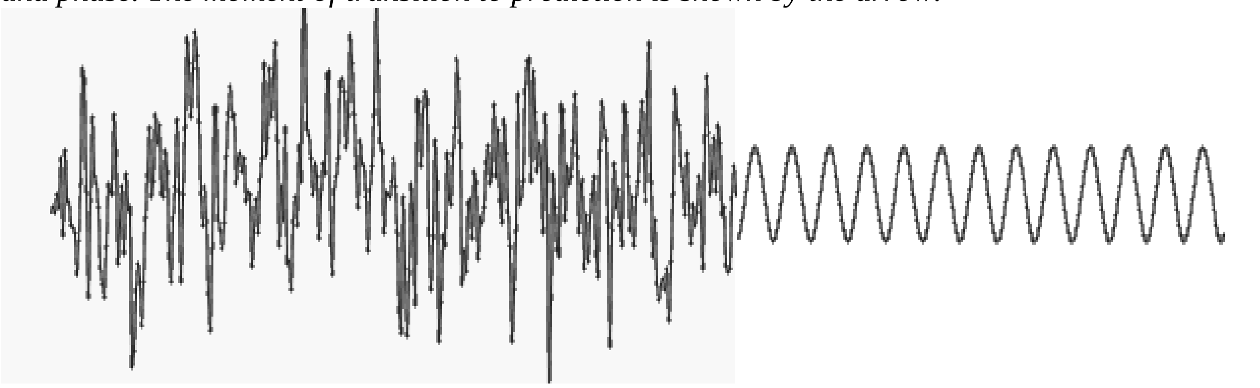

Fig.10. Mixture of the same signal with Gaussian noise. Detection and isolation of a single monochromatic component dominating on the spectrum is ensured almost without distortion with a very small frequency error.

After obviously correct illustrations, the subsequent verification should include an accurate (!) prediction of the Kelvin signal, otherwise all previous reasoning is cheap and can be ignored.

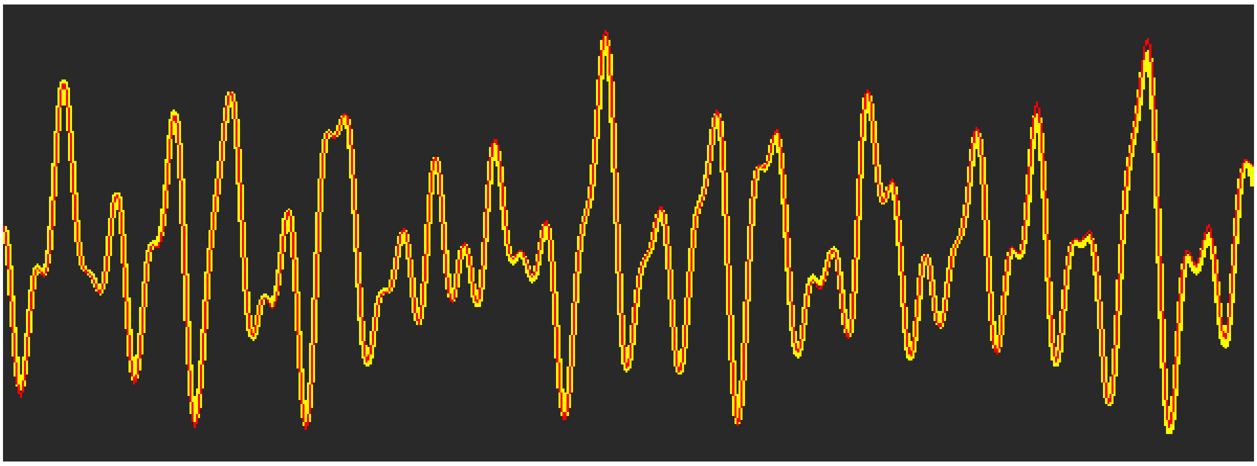

Fig.11. Reverse Kelvin noise prediction: the yellow line indicates the real signal, the thin red line the predicted values. The right part of the graph shows a slight degradation of the match (due to errors in the spectroanalyzer data window and incomplete side lobe suppression). The sweep speed and gain are increased relative to the previous illustrations for clarity.

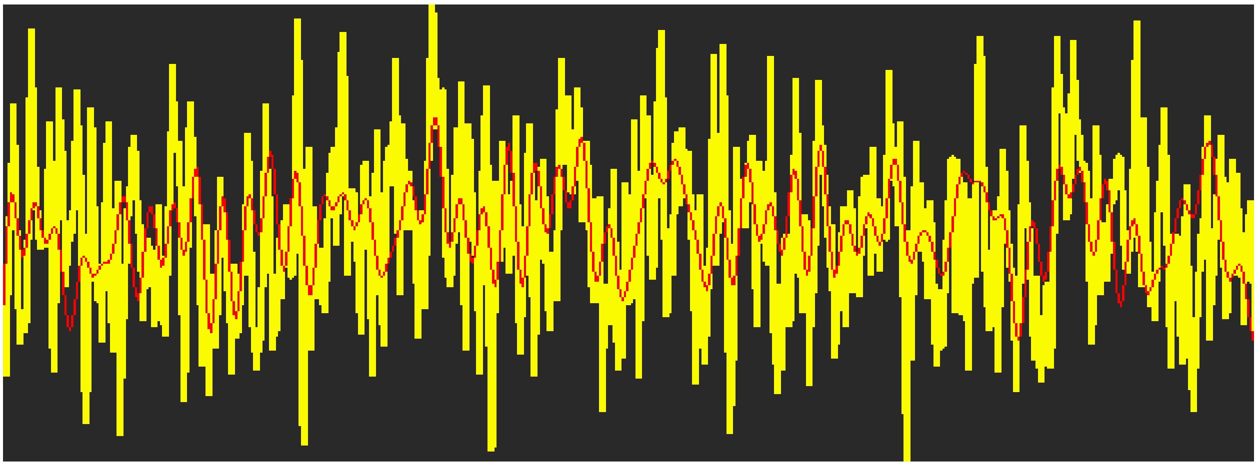

Fig.12. Reverse prediction of band-limited Gaussian noise. Clear impossibility of the problem: only the low-frequency components of the oscillations are tracked to some extent. The high-frequency part of the signal goes into the "spectrum masking" mode also due to violation of the Kotelnikov condition.

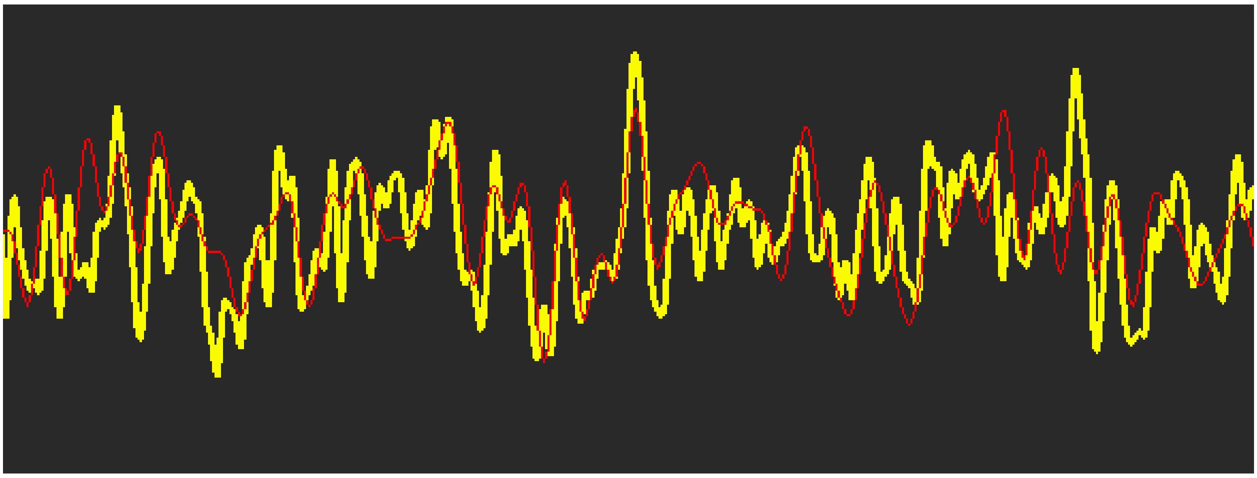

Fig.13. Additional Gaussian noise bandwidth constraint from above. Prediction is still problematic, although the mismatch between the input and predicted signals is noticeably reduced.

Two preliminary conclusions can be drawn from the results of the experiments.

1. The Kelvin noise process is well predictable, at least on a short realization.
2. The Gaussian process cannot be effectively predicted even under the condition of bandwidth limitation from above.

The question naturally arises: why these prediction exercises are necessary if the existence of Kelvin noise is not generally accepted up to now, and perhaps illustrates only the author's obsession (which should never be ruled out a priori). In justification I will quote: "The attribution of a process to the class of random ones can be conditioned either by its physical nature or by the conditions of its study leading to the insufficiency of a priori data [6]".

And let us recall, for example, that there are many different internal combustion engines operating in our environment. Almost all of them have at their output a rotational motion with a stable or relatively slowly changing frequency. The vibrations produced by this motion have many even and odd harmonics due to vibrations due to uncompensated moments of inertia as well as to valve train and power take-off drives. In addition to the harmonics, unsynchronized noise is also generated. The resulting acoustic signal has both a predictable component consisting of noise and vibration synchronized with the main harmonic and an unpredictable “purely Gaussian” part. This can provide an opportunity to assess the technical condition of the engine, since the recorded realization of the engine noise can be predicted in inverse time.

The result will be two realizations of the signal: the original noise and the result of its prediction. By calculating the R(p) (normalized mutual correlation) coefficient between these two sequences, we can estimate the contribution of the Kelvin component to the total noise.

And if, for example, the correlation coefficient increases during operation, it is a sure sign of wear or damage to parts traveling synchronously with the fundamental harmonics. If the correlation coefficient decreases (the unpredictable component grows), then somewhere there is increased friction, skipping of gases, etc. Such a method can be especially useful for continuous cycle turbines - constant non-contact control is not superfluous.

Of course, instead of normalized correlation you can calculate the mismatch energy, or in general use the whole set of mathematical statistics tools. But as it seems to me, it is unlikely to get much additional information regarding correlation analysis.

Let's assume that the parameter corresponding to the value of R(p), i.e. the “signal predictability coefficient”, can find application in a wide variety of fields. Even a stockbroker or currency speculator would be interested in having a prediction for the probability of winning based on the previous behavior of the price curve. Such fields as electroencephalography may not be mentioned: obtaining a map of anomalies of the brain rhythm population will undoubtedly increase the level of diagnostics. Especially in those cases where the EEG changes are implicit, in a way that only very experienced specialists allow themselves to assume pathology.

Probably, the assessment of signal predictability can also be used for studies in radio astronomy, which studies both purely stochastic signals and rhythmic parcels of various sources. Oh, and the “radio sky predictability map” would be worth looking at.

In addition, it would be good to know: is gravitational noise a Kelvin process, and are powerful gravitational waves different from it?

For this purpose, let us accept as an axiom the fundamental prohibition: “If signals exist simultaneously and their spectra overlap, a complete separation of signals is impossible”. We also take into account that the gravitational wave registered by detectors always exists simultaneously with gravitational (and not only) noise. Of course, it will not be possible to separate these two signals completely; spectral whitening is asked not to propose for an obvious reason: the existence of an a priori hypothesis. Naturally, the classical correlation method, which is sensitive to the phase of the analyzed processes, is not suitable either. But if we calculate the predictability coefficient of the signal, then the described obstacles can be bypassed without violating the basic provisions of information theory. Let us try to do exactly this, and start with the control of the filtering algorithm.

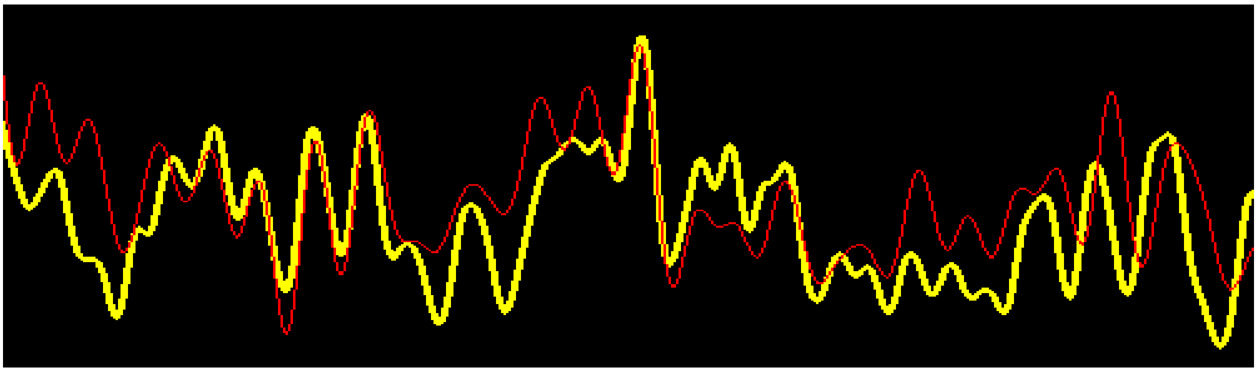

Fig.14. Gaussian noise reference signal after passing a filter system identical to the one used for gravitational wave extraction. Reverse prediction. R(p) = 0.5035

Let us use the records of the LIGO project and do not forget that the extraction of a useful signal from the most diverse noise occurred at the limit of the hardware capabilities. Naturally, the “noisy” signal will have to show less predictability.

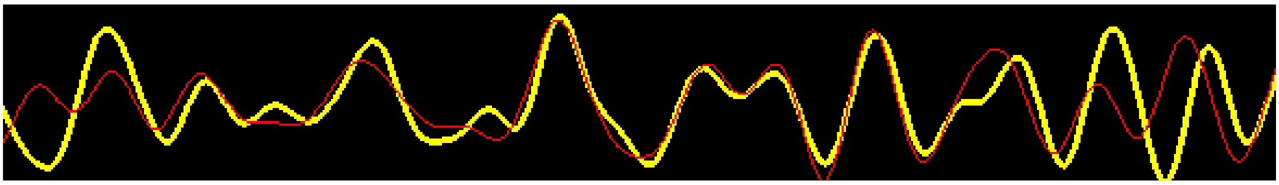

Fig.15. “Quiet” interference-free section of the gravitational background recording. R(p) =0 .6608

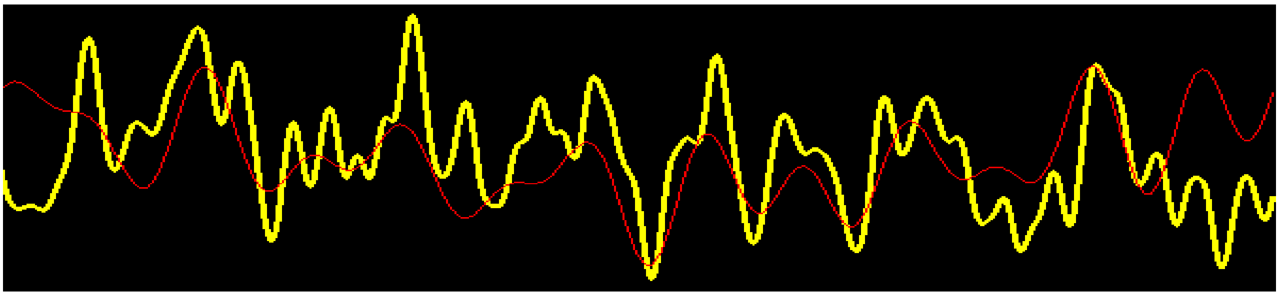

Fig.16. A section of the recording with some external interference. R(p) = 0.3162

The result is quite expected: the more noise and interference, the lower the R(p) coefficient. Now let us try to predict the officially confirmed gravitational wave. We note once again that its extraction [7] was made from unfiltered LIGO records kindly provided for general use.

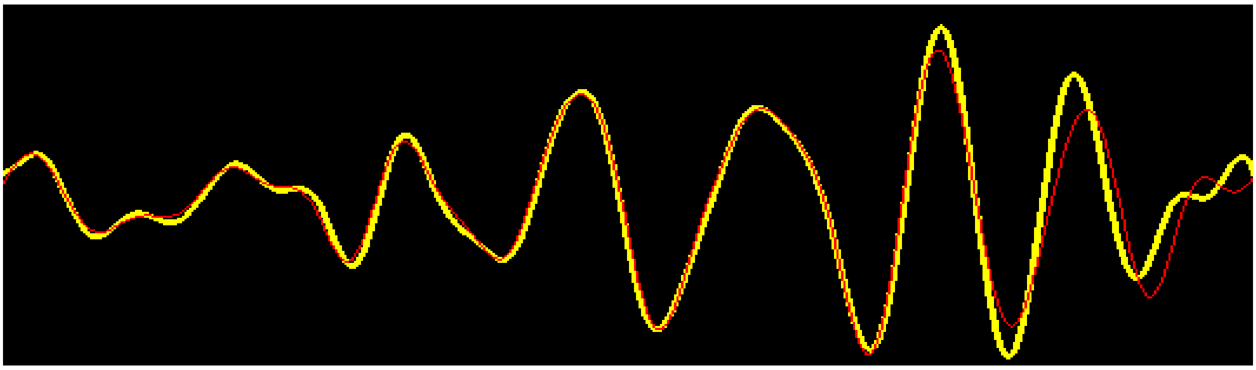

Fig.17. Reverse prediction of the gravitational wave extracted from the LIGO laboratory record using matched FIR filtering without using (NB!) the spectral whitening method. The signal predictability coefficient R(p) = 0.9587 (!!!)

The obtained value of the coefficient is expectedly high: after all, the initial object was a system of masses rotating with increasing angular velocity. And this rotation provides the main cosine component in the signal registered at distances much larger than the rotation radius.

The high predictability of the gravitational background is also not surprising: indeed, there are many rotating masses in the zone of space from which a sufficiently powerful signal can come. And these oscillators should form exactly the Kelvin noise - the additive mixture of signals of rotating masses quite corresponds to his system of “...flywheels, bars, springs”. However, such hypotheses in the field of gravitational astronomy are by no means welcomed by established specialists in this field.

The main thing is that R(p) really works, and the results are quite consistent with expectations. It can be used in practice. Shall we make the stock exchange players happy?

P.S. Pearson's algorithm is used to calculate R(p). It has one unpleasant feature. If “0” appears in the sub-root expression, it can be divided by it with corresponding consequences. This disadvantage can be eliminated by forcibly adding one unit of the lowest digit to the sub-root expression, the accuracy of calculations practically does not suffer.

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